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Teaching Electromagnetic Wave Simulation: Applying Perfectly Matched Layer (PML) in Two-Dimensional Fields

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Abstract

Electromagnetic waves play a crucial role in various modern technological applications, such as wireless communication, radar, and signal processing. A deep understanding of the behavior of these waves in different media and boundary conditions can be achieved through numerical simulation. An in-depth understanding of electromagnetic wave behavior is essential in science and engineering education, in particular to help students visualize and analyze complex phenomena. This study discusses the two-dimensional (2D) simulation of electromagnetic waves using the Finite Difference Time-Domain (FDTD) method, with the implementation of Perfectly Matched Layer (PML) to address wave reflections at the simulation domain boundaries. Fundamental electromagnetic laws, such as Gauss's Law, Faraday's Law, and Ampere's Law, are used as the basis to develop the curl Maxwell equations in 2D. The proposed simulation algorithm calculates the electric fields (Ex, Ey) and magnetic field (Hz) considering the PML damping factor. Simulations are performed with various PML constant values (0.00001, 0.00005, 0.0001) to observe the effectiveness of wave absorption. The simulation results show that increasing the PML constant value reduces reflections and improves the accuracy of the simulation results. Therefore, the FDTD method integrated with PML is effective in modeling the behavior of electromagnetic waves in a 2D domain, providing more realistic and accurate results.

Keywords: electromagnetic waves propagation, electromagnetic waves simulation, perfectly matched layer (PML)

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INTRODUCTION

Understanding electromagnetic waves is an important part of science and engineering education (Wong & Lim, 2020). However, students often face difficulties in visualizing and understanding the concepts due to their abstract nature (Bouchée et al., 2021). To overcome such challenges, simulation of electromagnetic waves in a two-dimensional plane has emerged as an effective tool, as it allows modeling of complex physical phenomena in an easier way, this allows students to visualize and better understand the behavior of electromagnetic waves in various contexts (Weichman et al., 2024). However, 2D simulations face significant challenges, especially with spurious reflections appearing at the boundaries of the simulation domain, which can compromise the accuracy of the results (Antony et al., 2024). The development of the Perfectly Matched Layer (PML) has addressed this challenge by minimizing such reflections and improving simulation accuracy (Du & Zhang, 2023). Despite the effectiveness of PML, previous research still leaves gaps that need to be addressed, especially in terms of implementing and optimizing PML across different simulation configurations.

Electromagnetic waves are a fundamental physical phenomenon and have various important applications in modern technology, including wireless communication, radar, and signal processing (Raya et al., 2024; Wu et al., 2023). The simulation of electromagnetic waves in two dimensions (2D) is an effective tool for understanding the behavior of these waves in various media and boundary conditions (Duhamel, 2024). The ability to simulate electromagnetic waves with high accuracy is crucial for developing more efficient and reliable devices and systems (Guo et al., 2021). Electromagnetic waves are waves formed by the oscillation of electric and magnetic fields that are perpendicular to each other and propagate through a vacuum or other mediums. Electromagnetic waves do not require a medium to travel, unlike mechanical waves such as water waves or sound waves, which need a medium to propagate. (Hayt et al. 2020). There are four fundamental laws of physics that form the basis of electromagnetic wave theory:

Gauss's Law for Electric Fields

Gauss's law states that the total electric flux passing through a closed surface is proportional to the total electric charge enclosed by that surface. Mathematically, Gauss's law can be expressed as:

$$\oint E \bullet dA = Q/\varepsilon_0 \quad (1)$$

with E is the electric field, dA is the surface area element, Q is the total electric charge enclosed by the surface, and ε_0 is the permittivity of free space. Gauss's Law can be used to calculate the electric field generated by a distribution of electric charge (Ishimaru, 2017)

Gauss's Law for Magnetic Fields

Gauss's Law for magnetic fields states that the total magnetic flux passing through a closed surface is always equal to zero. Mathematically, Gauss's Law for magnetic fields can be expressed as:

$$\oint B \bullet dA = 0 \quad (2)$$

with B is the magnetic field, dA is the surface area element, Gauss's law for magnetic fields indicates that there are no monopole sources of magnetic fields (Lee, 2018).

Faraday's Law

Faraday's Law states that the electromotive force (emf) induced in a closed circuit is proportional to the rate of change of the magnetic flux passing through that circuit. Mathematically, Faraday's law can be expressed as:

$$\varepsilon = -\frac{d\Phi}{dt} \quad (3)$$

where:
ε is the induced electromotive force
Φ is the magnetic flux
t is time
Faraday's law states that a change in the magnetic field can induce an electric field. (Giancoli, 2016).

Ampere's Law

Ampere's Law states that the line integral of the magnetic field along a closed path is proportional to the total electric current enclosed by that path. Mathematically, Ampere's law can be expressed as:

$$\oint B \bullet dl = \mu_0 I \quad (4)$$

with B is the magnetic field, dl is the differential length element of the path, μ_0 is the permeability of free space, I is the total electric current enclosed by the path. Ampere's Law shows that electric current can generate a magnetic field (Giancoli, 2016).

Implementation of Electromagnetic Waves in Computer Programs

Electromagnetic waves can be simulated in computer programs using various numerical methods to solve Maxwell's equations. Maxwell's equations are a set of four partial differential equations that underlie all electromagnetic phenomena. (John, 2019).

Electromagnetic wave simulation can be conducted for various fields, namely 1D, 2D, and 3D. Each dimension has different levels of complexity and suitability for different types of cases. 1D simulation is used for simple cases, such as wave propagation in cables. In this simulation, only one dimension is considered, which is the length of the cable. The method commonly used for 1D simulation is the Transmission Line Method (TLM). TLM simulates the cable as a connected series of resistors and inductors, and can be used to calculate impedance, reflectance, and wave transmission in the cable. (Colas et al., 2019). 2D simulations are used for more complex cases, such as wave propagation in antennas. In this simulation, two dimensions are considered, namely the length and width of the antenna. The commonly used methods for 2D simulations are the Finite Element Method, Finite Difference Method, and the Time Domain Finite Difference Method. (Finite Difference Time-Domain). FEM, FDM, and FDTD divide space into a grid consisting of small elements and calculate the values of the electromagnetic field in each element. FDTD is more accurate than FEM and FDM, but it is more complex to implement. Then, 3D simulations are used for the most complex cases, such as wave propagation in complex structures. In this simulation, three dimensions are considered: length, width, and height of the structure. The commonly used method for 3D simulation is the Finite Difference Time-Domain (FDTD) method. FDTD calculates the electromagnetic field values at each point in the grid directly and can be used for complex simulations with intricate geometries (Liu et al., 2023). In this simulation, we use the FDTD (Finite Difference Time-Domain) method in a two-dimensional simulation. FTDT can be used to simulate various electromagnetic phenomena, such as calculating antenna radiation patterns, studying how electromagnetic waves interact with objects, and assisting in the design of electromagnetic devices like filters, amplifiers, and integrated circuits. The advantages of FTDT are computational efficiency, simplicity, and accuracy for various complex cases. (Zhang, Y. X, 2019)

PML (Perfectly Matched Layer) and Its Influence on 2D Electromagnetic Wave Simulation

One of the main challenges in electromagnetic wave simulation is handling the boundaries of the simulation domain. Reflection of waves at the domain boundary can cause significant distortion, reducing the accuracy of the simulation. To address this issue, a layer known as the Perfectly Matched Layer (PML) is applied. PML is a numerical technique designed to absorb electromagnetic waves arriving at the boundaries of the simulation domain, thereby reducing reflections and allowing for more realistic simulations. PML is based on the principles of coordinate transformation and the modification of Maxwell's equations, according to Zhang (2019). Coordinate transformation maps physical space to complex space, where EM waves propagate exponentially. The modification of Maxwell's equations introduces complex losses that gradually increase with distance from the computational domain boundary, effectively absorbing the EM waves that propagate out of the domain (Colas et al., 2019).

In this study, the computational model used for the two-dimensional (2D) fields consists of a rectangular domain with dimensions of X by Y. The PML settings were carefully configured to minimize reflections at the simulation boundaries. The PML was implemented with a thickness of N grid cells, covering a physical distance of d units. A quadratic absorption profile was applied to ensure smooth attenuation of the electromagnetic waves as they enter the PML region. The reflection coefficient at the interface was set to R, ensuring that less than X% of the incident wave energy was reflected back into the computational domain. The boundary conditions at the PML interface were optimized to further reduce any residual reflections and maintain simulation accuracy.

When PML is applied to 2D EM simulations, the EM waves propagating out of the computational domain are effectively absorbed, thereby preventing reflections and producing more accurate simulation results. Without PML, wave reflections from the boundaries of the computational domain can cause interference and distortion in the electromagnetic field, leading to unrealistic simulation results (Varalakshmi et al., 2021). In summary, the chosen thickness and absorption profile of the PML, along with the specific material properties and wave source parameters, were validated through a series of test simulations, confirming their effectiveness in producing reliable and consistent results.

METHOD

Mathematical Formula

Maxwell's curl equations

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu}\nabla \times E - \frac{1}{\mu}\sigma^* H \quad (5)$$

$$\frac{\partial E}{\partial t} = -\frac{1}{\mu}\nabla \times H - \frac{1}{\epsilon}\sigma H \quad (6)$$

μ_0 : Vacuum permeability or free space permeability.
ϵ_0 : Vacuum permittivity or free space permittivity.
σ : Electrical conductivity

Maxwell's curl equations in a 2D field.

$$\sigma E_x + \epsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial H_y} \quad (7)$$

$$\sigma E_y + \epsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial H_x} \quad (8)$$

$$-\sigma_m H_z - \mu \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \quad (9)$$

(Mo, G., 2023)

Determinants of PML:

$$factor \; = e^{-\alpha*(\omega - i)} \quad (10)$$

(Ra'di, et al., 2015)

α = PML constant.
ω = thickness of PML
$\boldsymbol{i}$ = the distance from the beginning of the PML layer
(Oskooi, et al., 2010)
.
Wave transformation with PML factor

$$E_x(t+\Delta t) = factor \times E_x(t) + \frac{\Delta t}{\epsilon_0}(-\sigma E_x(t) + \frac{H_z(t+\Delta t) - H_z(t)}{dy}) \quad (11)$$

$$E_y(t+\Delta t) = factor \times E_y(t) + \frac{\Delta t}{\epsilon_0}(-\sigma E_y(t) + \frac{H_z(t+\Delta t) - H_z(t)}{dy}) \quad (12)$$

$$H_z(t+\Delta t) = factor \times H_z(t) + \frac{\Delta t}{\epsilon_0}(-\sigma H_z(t) + \frac{E_x(t) - E_x(t-\Delta t)}{dy} - \frac{E_y(t) - E_y(t-\Delta t)}{dx}) \quad (13)$$

(Amat, S. et al., 2017)

Algoritma

Variable Initialization
lx, ly = Length and width of the domain
epsilon0 = Permittivity of free space
mu0 = Permeability of free space
sigma = Damping coefficient
ny = Number of grid points in the y coordinate
nt = Number of iterations
nvis = Time interval for saving visualization data

```
pml_alpha = PML damping coefficient
pml_width = Width of the PML layer
dx = x coordinate step
dy = y coordinate step
dt = time step

# Grid initialization
nx = Number of grid points in the x coordinate
nx_pml = Number of grid points in the x coordinate PML
ny_pml = Number of grid points in the y coordinate PML

# Electric field in the x direction
Ex = [[0.0] * (ny_pml + 1) for _ in range(nx_pml)]

# Electric field in the y direction
Ey = [[0.0] * ny_pml for _ in range(nx_pml + 1)]

# Grid point coordinates in the x direction
xc = [-lx / 2 + dx / 2 + i * dx for i in range(nx_pml)]

# Grid point coordinates in the y direction
yc = [-ly / 2 + dy / 2 + i * dy for i in range(ny_pml)]

# Magnetic field
Hz = [[np.exp(-x**2 - y**2) for y in yc] for x in xc]

#Initialization of empty lists
Ex_list, Ey_list, Hz_list = [], [], []

# Main Iteration
for it in range(nt):
    # Update Ex
    for i in range(1, nx_pml):
        for j in range(1, ny_pml):
            Ex[i][j] += dt / epsilon0 * (-sigma * Ex[i][j] + (Hz[i][j+1] - Hz[i][j]) / dy)

    # Update Ey
    for i in range(1, nx_pml):
        for j in range(1, ny_pml):
            Ey[i][j] += dt / epsilon0 * (-sigma * Ey[i][j] - (Hz[i+1][j] - Hz[i][j]) / dx)

    # Update PML koordinat x
    for i in range(pml_width):
        factor = np.exp(-(pml_width - i) * pml_alpha)
        for j in range(ny_pml):
```

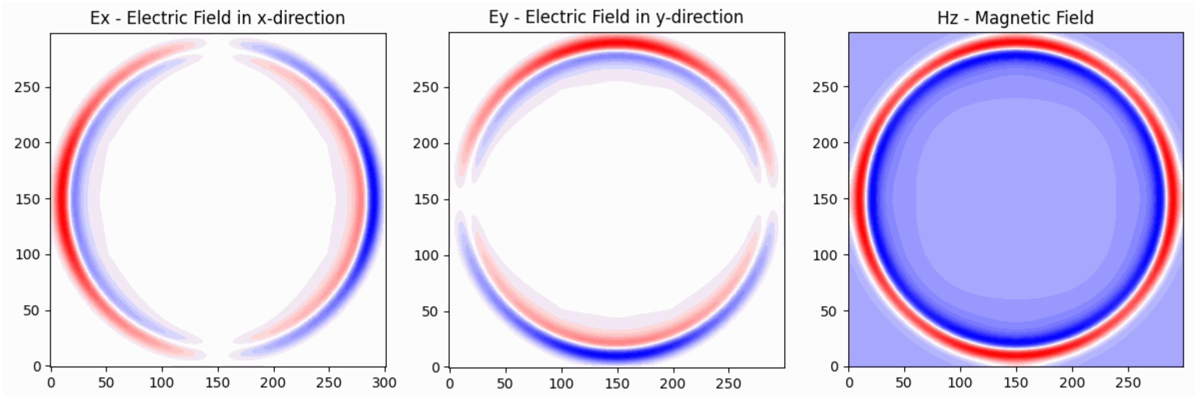
```
            Ex[i][j] *= factor
            Ex[nx_pml - i - 1][j] *= factor

    # Update PML koordinat y
    for i in range(pml_width):
        factor = np.exp(-(pml_width - i) * pml_alpha)
        for j in range(nx_pml):
            Ey[j][i] *= factor
            Ey[j][ny_pml - i - 1] *= factor

    # Update Hz
    for i in range(1, nx_pml):
        for j in range(1, ny_pml):
            Hz[i][j] += dt / mu0 * (-sigma * Hz[i][j] + (Ex[i][j] - Ex[i][j-1]) / dy - (Ey[i][j] - Ey[i-
1][j]) / dx)

    # Menyimpan data
    if it % nvis == 0:
        Ex_list.append([row[:] for row in Ex])
        Ey_list.append([row[:] for row in Ey])
        Hz_list.append([row[:] for row in Hz])
```

RESULTS AND DISCUSSION

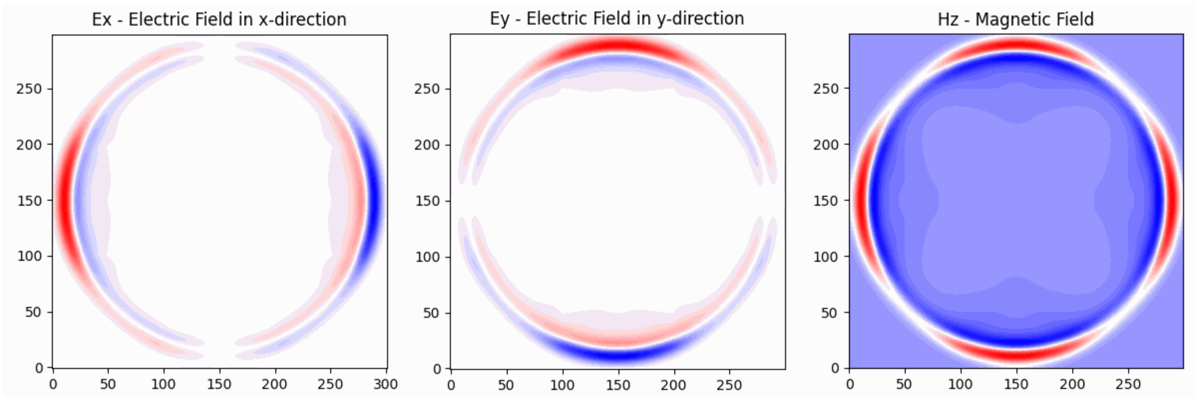
To determine the results of the simulation from the program that has been created, a simulation of electromagnetic waves in two-dimensional space was conducted by running the simulation with different values of the PML constant variable. Perfectly Matched Layer (PML) is a technique used in numerical wave simulation to eliminate reflections from the boundaries of the simulation domain. Several repetitions were carried out to demonstrate the significant influence of PML on the results of the generated wave simulations. The repetition was carried out by running the program with the variable pml set to 0.00001, 0.00005, and 0.0001.

The fixed variable parameters used in the simulation code above include lx, ly = 40.0, 40.0; epsilon 0 = 1.0; mu0 = 1.0; sigma = 1.0; pml width = 100; number of iterations = 7000; and ny = 100. Based on the simulation results, it can be analyzed how the wave propagation occurs when entering different PMLlayers.

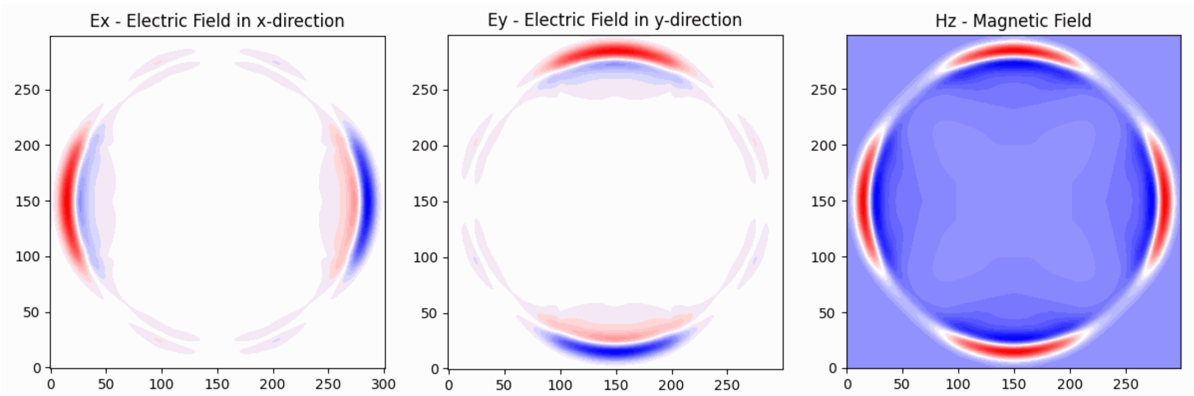
The first graph depicts the propagation of the electromagnetic wave Ex along the x-axis. The second graph illustrates the propagation of the electromagnetic wave Ey along the y-axis. The third graph shows the propagation of the electromagnetic wave Ez along the z-axis. The red color in the graphs represents the direction of the vector moving outward, while the blue color indicates the direction of the vector moving inward. Figures 1.a, 1.b, and 1.c show how the wave propagation results after entering the pml region at the same time. Figure 1.a illustrates the simulation results of wave propagation upon entering the pml with a constant of 0.00001. The simulation results show that there are only slight indentations at the corners of the coordinate points where the PML layer is located. In Figure 1.b, it is clearer that the waves entering the corners of the PML layer begin to experience significant absorption compared to the previous experiment with a PML constant of 0.00005. The same occurs when the PML constant is increased again; Figure 1.c shows the results of wave absorption at the corners of the PML layer that are more apparent. This happens because the PML constant provided is larger than before, which is 0.0001.



(a) pml = 0.00001



(b) pml = 0.00005



(c) pml = 0.0001

Figure 1. (Results of the simulation with different PML variables.)

CONCLUSION

The propagation of electromagnetic waves can be simulated in a computer program using one of the numerical methods, namely Finite Difference Time-Domain (FDTD). The FDTD method is particularly effective in simulating electromagnetic waves in two-dimensional fields, offering a computational process that is efficient, simple, and accurate. In this study, wave simulation using FDTD was integrated with the Perfectly Matched Layer (PML) technique to eliminate reflections from the boundaries of the simulation domain. The simulation results demonstrate that as the PML constant increases, the absorption of the electromagnetic waves entering the PML layers becomes more pronounced, effectively reducing boundary reflections and enhancing the overall accuracy of the simulation. When comparing these findings with previous studies, such as those by Zhang (2019) and Varalakshmi et al. (2021), our results confirm that PML is a highly effective method for minimizing unwanted reflections in computational domains. Our study further advances this understanding by optimizing PML settings specifically for 2D simulations, addressing challenges unique to this context. Unlike earlier research that primarily focused on three-dimensional domains, our work provides a detailed analysis of PML's impact in a 2D setting, offering new insights into its application and effectiveness.

The improved accuracy of simulations achieved through the integration of PML has significant implications for educational settings. By providing a more precise representation of electromagnetic wave behavior, students can gain a clearer understanding of complex concepts in electromagnetic theory. The enhanced visualization and reduced distortions in the simulations make them a powerful tool for teaching, allowing students to engage more deeply with the material and bridge the gap between theoretical knowledge and practical application..

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