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Enhancing Student Understanding of Heat Distribution in Metal Rods Through Interactive Learning Using Finite Difference Methods

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Abstract

This study discusses the comparison of two finite difference methods, namely the explicit method and the Crank-Nicolson method (implicit), in simulating heat propagation in a metal rod. Heating is done by lighting a candle under the metal rod which is then extinguished after some time. This research aims to improve students' understanding of heat distribution in metal rods through an interactive method based on the Finite Difference Method, which is also expected to improve the ability to analyze and apply physics concepts in a practical context. The simulation results show that the explicit method requires very small time steps to achieve good stability, resulting in longer computation times. On the other hand, the Crank-Nicolson method demonstrates better and more consistent numerical stability, even with larger time intervals. Experimental modifications with varying time intervals show that the Crank-Nicolson method remains stable and provides more accurate results compared to the explicit method. Therefore, the Crank-Nicolson method is more recommended for long-term simulations requiring high stability and accuracy.

Keywords: crank-nicolson, diffusion equation, explicit method, finite difference, implicit method, thermal diffusion

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INTRODUCTION

Computation is the process of finding a solution to a problem expressed in a mathematical model (Zhang & Yang, 2024). One application of computing can be found in the study of physics problems, which are usually discussed in the field of computational physics. Computational physics involves the combination of physical phenomena based on the principles of the laws of physics, numerical methods, and computer programming (Weller et al., 2022). Through computational physics, we can solve various complex physics problems, including problems involving partial differential equations (Jung et al., 2024).

Differential equations that contain partial derivatives are called partial differential equations (PDP) (Li & Carvalho, 2024). Simply put, a partial differential equation is an equation that contains partial derivatives of an unknown function. This is different from ordinary differential equations, where the unknown function depends on one variable and all derivatives are ordinary derivatives. Many algorithms used for numerical simulation of physics problems solve discrete approximations of partial differential equations (PDPs). These PDPs are derived in the framework of differential calculus and can be formulated in terms of coordinate-invariant first-order differential operators such as the gradient of a scalar or vector, the divergence of a vector or tensor, and the curl of a vector. PDPs express fundamental physical laws such as conservation of mass, momentum, and total energy in fluid flow, or Faraday, Maxwell-Ampére, and Gauss laws in electromagnetism (Yong, 2020).

In partial differential equations, the unknown function u, or dependent variable, depends on two or more independent variables. Usually in describing natural phenomena the dependent variable u will depend on one or more space variables x,y,z and time t. Sometimes the dependent variable depends on space variables only. (Farlow, 1994). In his book also Farlow (1994) mentioned, there are four common partial differential equations. Among them are as follows:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad (One - dimensional\ heat\ equation) \tag{1}$$

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2} \quad (One - dimensional\ wave\ equation \tag{2}$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (Laplace\ equation\ in\ polar\ coordinates) \tag{3}$$

$$\frac{\partial^2 U}{\partial t^2} = \alpha^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \quad (Two - dimensional\ wave\ equation) \tag{4}$$

Of the four partial differential equations above, one of them will be discussed, namely the one-dimensional heat equation or it can be called thermal diffusion. The diffusion equation is a linear partial differential equation that represents the movement of a part from a high concentration to a low concentration part (Kalogeris & Papadopoulos, 2021). Heat transfer always occurs from higher temperatures to lower temperatures as described by the second law of thermodynamics (Dowling et al., 2020). The general formula of the thermal diffusion equation is expressed as follows :

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{5}$$

where the parameter α is called thermal diffusivity, and its value depends on the type of metal rod. This equation is also called the parabolic partial differential equation (Farlow, 1994).

Parabolic partial differential equations are a type of equation used to model various physical phenomena such as heat diffusion, fluid flow, and stochastic processes. Parabolic partial differential equations are mostly one-dimensional heat equations (conduction equations) (Sofiani, 2023). These equations have a general form that includes both time and space derivative components, and are used to analyze the dynamics of systems that change over time. Many researchers have worked on the famous parabolic partial differential equation (one-dimensional heat conduction equation) using various numerical methods but of all the numerical methods the most widely used is the finite difference method (Johnson & Oluwaseun, 2020). There are many types of finite difference approaches used to solve the heat equation. There are two finite difference methods that can be used to solve the one-dimensional heat equation, namely the explicit finite difference method and the implicit finite difference method (Lang & Schmitt, 2023).

Teaching heat distribution has its own challenges as it involves abstract concepts that are difficult for students to visualize (Mitropoulos et al., 2023). As a result, they may have difficulty in linking theory to practical applications, which can hinder their deep understanding and ability to apply these concepts in real situations. Using traditional methods often relying on theoretical explanations and mathematical formulas, can fail in helping students understand the dynamic nature of heat flow and its dependence on variables such as temperature, time, and material properties (Yao et al., 2022).

METHOD

Learning Design

This learning is designed to enhance students' understanding of the concept of heat distribution in rods. Learning is structured based on the steps in calculating equations up to the steps of developing learning using simulations. This approach ensures that learners can firmly develop the foundation of a concept before moving on to more complex topics or concepts. Each student has a different learning style; therefore, the learning developed contains various interactive elements to clarify concepts both visually and mathematically. In understanding a concept, there are times when the approach to learning can provide benefits in terms of collaborative learning, where students can work individually or in teams to solve a given problem. Thus, it can indirectly foster communication skills and the ability to collaborate effectively, which are useful for deepening students' understanding of a concept.

Explicit Method

The finite difference explicit method is one of the numerical approaches used to solve partial differential equations such as the diffusion equation. This method approximates the derivative in the partial differential equation by using forward differences or backward differences in time, as well as center differences in space.

In the context of heating a metal rod with the diffusion equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ the explicit finite difference method replaces the first derivative with a forward or backward difference in time, and the second derivative with a center difference in space. Mathematically, this can be represented as :

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$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}}{\Delta x^2}$$

A forward finite difference approach is used to approximate the time derivative as follows :

$$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^{l}}{\Delta t}$$

Substituting Equations (6) and (7) into Equation (5) yields:

$$\frac{T_i^{l+1} - T_i^{l}}{\Delta t} = \alpha \frac{T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}}{\Delta x^2}$$

Which can be solved

$$T_i^{l+1} = T_i^{l} + \lambda\left(T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}\right)$$

where $\lambda = \frac{\alpha \Delta t}{\Delta x^2}$ is called the mesh ratio parameter (Chapra & Canale, 2010).

In equation (9) T_i^{l+1} is expressed explicitly in the form T_{i-1}^{l}, T_i^{l} and T_{i+1}^{l}. Therefore, this formula is called an explicit formula for solving the one-dimensional heat equation. It can be shown that Equation (9) is valid only for $0 \leq \lambda \leq \frac{1}{2}$, which is referred to as the stability condition for the explicit formula. Using the initial conditions in Equation (9), we get a system of linear equations for n = 0,1,2.... These linear equations are solved to obtain a new time step solution. If we fix $\lambda = \frac{1}{2}$ in equation (9), we obtain a simple formula:

$$T_i^{l+1} = \frac{1}{2}\left(T_{i+1}^{n} + T_{i-1}^{n}\right)$$

Implicit Method

The implicit finite difference method is a numerical technique used to solve partial differential equations (PDPs). Unlike explicit methods, it involves solving a linear system of equations at each time step, which allows for better stability even with larger time steps. In the implicit approach, the solution to a set of finite element equations involves iteration until a convergence criterion is met for each step. The word 'implicit' in this paper refers to a method in which the state of the finite element model is updated from time t to $t + \Delta t$. A fully implicit procedure means that the state at $t + \Delta t$ is determined based on information at time $t + \Delta t$, while explicit methods solve for $t + \Delta t$ based on information at time t (Harewood & McHugh, 2007).

This method approximates partial derivatives in an implicit way, where the values at the next time $(n+1)$ are used in the current calculation (n). For example, for the diffusion equation, this method often uses the Crank-Nicolson scheme. The Crank-Nicolson scheme is one of the development schemes of the explicit and implicit schemes, which is the average value of the two methods. However, the form of the Crank-Nicolson scheme is an implicit scheme. The advantage of this method compared to other finite difference methods is that it is unconditionally stable. In the Crank-Nicolson scheme, the differential with respect to time t is written in the form of a forward difference (Msmali et al., 2021).

The Crank-Nicolson method provides an alternative implicit scheme that has second-order accuracy in space and time. To provide this accuracy, a difference approach is developed at the midpoint of the time step. To do this, the first derivative of time can be approximated at $t^{l+1/2}$ by:

$$\frac{\partial T}{\partial t} \cong \frac{T_i^{l+1} - T_i^l}{\Delta t}$$

The second derivative in space can be determined at the midpoint by averaging the difference approaches at the beginning (t^l) and end (t^{l+1}) of the time step:

$$\frac{\partial^2 T}{\partial x^2} \cong \frac{1}{2}\left[\frac{T_{i+1}^l - 2T_i^l + T_{i+1}^l}{(\Delta x)^2} + \frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{(\Delta x)^2}\right]$$

Substituting equations (11) and (12) into equation (5), yields:

$$-\lambda T_{i-1}^{l+1} + 2(1+\lambda)T_i^{l+1} - \lambda T_{i+1}^{l+1} = \lambda T_{i-1}^l + 2(1-\lambda)T_i^l + \lambda T_{i+1}^l$$

Where $\lambda = \frac{\alpha \Delta t}{(\Delta x)^2}$. As with the simple implicit approach, the boundary condition $T_0^{l+1} = f_0(t^{l+1})$ can be determined to derive versions of Equation (13) for the first and last interior nodes. For the first interior node :

$$2(1+\lambda)T_1^{l+1} - \lambda T_2^{l+1} = \lambda f_0(t^l) + 2(1-\lambda)T_1^l + \lambda T_2^l + \lambda f_0(t^{l+1})$$

and for the last interior node:

$$-\lambda T_{m-1}^{l+1} + 2(1+\lambda)T_m^{l+1} = \lambda f_{m+1}(t^l) + 2(1-\lambda)T_m^l + \lambda T_{m-1}^l + \lambda f_{m+1}(t^{l+1})$$

Although Equations (14) and (16) are slightly more complicated compared to the usual implicit Equations, they are tridiagonal and therefore efficient to solve.

The Crank-Nicolson method is often used to solve linear parabolic PDEs in one space dimension. Its advantages become more prominent for more complicated applications such as those involving unequally spaced meshes. The non-uniform mesh arrangement is often advantageous when we have prior knowledge that the solution varies rapidly at localized parts of the system (Chapra & Canale, 2010).

In the research of Panigrahi et al. (2019) there are four types of numerical methods that have been used for the prediction of various thermo-physical and mass transfer in grain storage, namely the finite difference method (FDM), finite element method (FEM), finite volume method (FVM) and discrete element method (DEM). In these methods, partial differential equations (PDEs) allocated to each layer, element or control volume undergo further simplification into discrete linear equations leading to convergent approximation of solutions at different nodes. Four different time discretization schemes are followed to predict the variation of temperature and moisture content with time namely Galerkin, Euler backward and forward stepped and Crank-Nicolson schemes. Among all mentioned stepped methods, Crank-Nicolson was found to be the best as it gives second level accuracy on time and was used for simulation. The equation discussed in the research of Sanjaya and Mungkasi (2017) is also a parabolic partial differential equation that can be solved effectively using the finite difference method. This method is based on direct discretization of the differential equation, which allows approaching the solution through numerical techniques. In particular, the solution of the parabolic partial differential equation shows continuity, even when the initial conditions are discontinuous (Sanjaya & Mungkasi, 2017).

Therefore, the purpose of this study is to explain the difference between implicit and explicit finite difference methods in solving the diffusion equation for a scenario of heating a metal rod in the center with constant end conditions. In this experiment, a homogeneous metal rod of length l is placed on top of a heat source right in the center, while insulators are placed at both ends to maintain zero temperature. The heating process is carried out by lighting a candle under the metal rod, which is then turned off after some time. An analysis was conducted to see the difference between the implicit and explicit finite difference approaches in determining the temperature distribution along the metal rod during the heating and cooling process. It is hoped that the results from this study can provide better insight in choosing the most suitable numerical method for simulating diffusion phenomena in cases like this.

COMPUTATIONAL METHOD

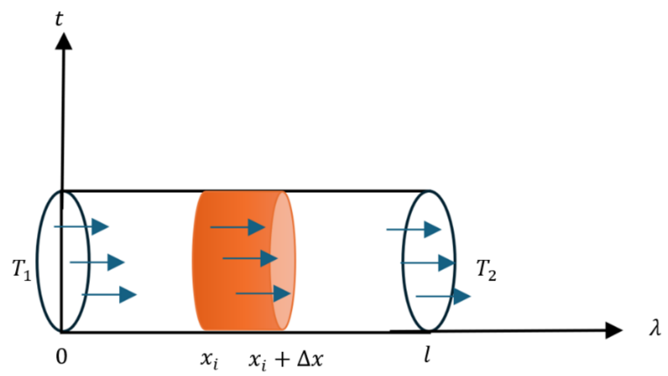

Figure 1. Heat propagation in Metal Rods

The case to be sought is the Comparison of Explicit and Implicit Finite Difference Schemes on the Diffusion Equation for Heating a Metal Rod in the Center with Constant Conditions at the Ends. Given a candle and a homogeneous metal rod of length l. The candle is placed under the metal rod right in the center, then given an insulating object placed at both ends. In this case, the insulator serves to maintain the temperature at both ends of the metal at zero degrees. After that, the candle is lit for some time, then the candle is turned off. For a clearer illustration, see Figure (2).

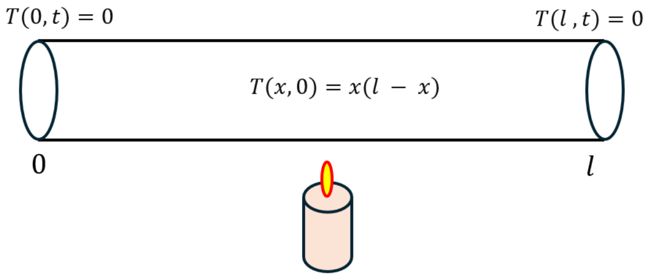

Figure 2. Illustration of heating a metal rod

The heat distribution illustrated in Figure (2) will be determined by equation (5) when $0 < x < l$, for every $t > 0$. Then given the initial value $T(x,0) = x(l - x), 0 < x < l$, with the boundary condition $T(0,t) = T(l\,,t) = 0$ for every $t > 0$. The problem will be solved using explicit and implicit finite difference methods.

Furthermore, a simulation of heat propagation on a heated metal rod is carried out. If the length of the metal rod is taken as $l = \pi = 3{,}14$ unit length, the domain $0 \le x \le 3{,}14$ is obtained In this case, heat propagation will be observed for 2 units of time, so the domain $0 \le t \le 2$ is obtained.

Algorithm

Explicit Method

1. Start
2. Initialize Parameters:L, T, α, dx, dt
   Calculate the number of space and time steps:
   2.1 Calculate the number of space steps Nx as int(L/dx)+1.
   2.2 Calculate the number of time steps Nt as int(T/dt)+1.
3. Position and Time Matrix :
   3.1 Create a linear array for position x from 0 to L with Nx steps.
   3.2 Create a linear array for time t from 0 to T with Nt steps.
4. Initialize the Temperature Matrix :
   4.1 Create a temperature matrix u of size Nt×Nx.
   4.2 Initialize all elements of the temperature matrix u with zero.
   4.3 Set the initial condition u[0,:] with the formula u(0,x)=x·(L−x)
5. Explicit Scheme:
   5.1 Calculate the value r as α·dt/dx^2
   5.2 For each time step n from 0 to Nt−2:
      5.2.1 For each position i from 1 to Nx−2:
         5.2.1.1 Calculate the value u[n+1,i] as u[n,i]+r·(u[n,i+1]−2·u[n,i]+u[n,i−1]).
6. Plotting Results:
   6.1 Plot the simulation results at certain times to see the heat propagation.
   6.2 Add labels, titles, and grids to the plots for easier interpretation.
7. Finish

Crank-Nicolson (Implicit) Method

1. Start
2. Initialize the parameters: L, T, α, dx, dt
3. Calculate the number of time and space steps :
   3.1 Calculate the number of space steps Nx as int(L/dx)+1.
   3.2 Calculate the number of time steps Nt as int(T/dt)+1.
4. Position and Time Matrix:
   4.1 Create a linear array for position x from 0 to L with Nx steps.
   4.2 Create a linear array for time t from 0 to T with Nt steps.
5. Initializing the Temperature Matrix :
   5.1 Create a temperature matrix u of size Nt×Nx.
   5.2 Initialize all elements of the temperature matrix u with zero.
   5.3 Set the initial conditions u[0,:] with the formula u(0,x)=x·(L-x).
6. Crank-Nicolson scheme:
   6.1 Calculate the value of r as α·dt/dx^2.
   6.2 Create matrices A and B:
      6.2.1 A=diag((1+2·r)·ones(Nx−2))+diag(−r·ones(Nx−3),k=1)+diag(−r·ones (Nx−3),k=−1)
      6.2.2 B=diag((1−2·r)·ones(Nx−2))+diag(r·ones(Nx−3),k=1)+diag(r·ones(Nx−3),k=−1)
7. Time Iteration:
   7.1 For each time step n from 0 to Nt−2:
      7.1.1 Calculate b from B·u[n,1:−1].
      7.1.2 Solve the linear equation A·u[n+1,1:−1]=bA·u[n+1,1:−1]=b to get the value of uu at the next time step.
8. Plot Results:
   8.1 Plot the simulation results at certain time steps to see the heat propagation.
   8.2 Add labels, titles, and grids to the plots for easier interpretation.
9. Finish

RESULTS AND DISCUSSION

Based on the experimental results, the following results are obtained. The space interval (dx) used is 0.01 and the time interval (dt) is 0,0001. Where the graph results produced by the explicit method are not perfect (can be seen in Figure 2). This is because in the explicit scheme, each time step is calculated directly from the values in the previous time step. In the explicit scheme, the value of $\lambda = \alpha \frac{\partial^2 t}{\partial x^2}$ must satisfy the Courant-Friedrichs-Lewy (CFL) stability condition for the solution to remain stable. When λ is greater than the stability constraint (generally $\leq \frac{1}{2}$), the solution may become unstable and produce oscillations or even divergence in the results. In this experiment, the value of $\lambda = 1$ exceeds the stability boundary, causing numerical instability and imperfect graphs.

Unlike the implicit method, which uses a time approximation that includes the values at the previous time step and the next time step, and therefore is not subject to strict stability constraints like the explicit scheme. The Crank-Nicolson method, which is an implicit method, has the advantage of better numerical stability. It is not subject to strict stability constraints and remains stable for all time intervals dt and space dx. This means that even when using a value of $dt = 0.0001$, Crank-Nicolson scheme produces stable and accurate solutions without experiencing the instability problems faced by explicit schemes. The following graphs show the results of the heat propagation of the metal rod by the Explicit and Implicit methods.

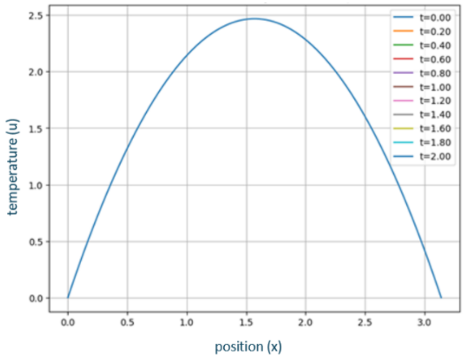

Figure 2. Heat propagation graph of metal rod by Explicit method with $dt = 0.0001$

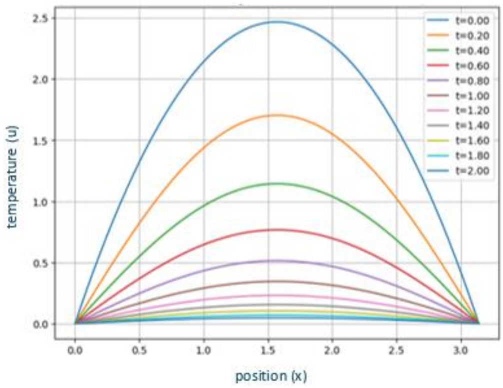

Figure 3. Heat propagation graph of metal rod by implicit method with $dt = 0.0001$

Then we tried changing the time interval (dt) by 0.00001 to see if the large value of the time interval affects the numerical calculation process of this Explicit method. The results are as follows.

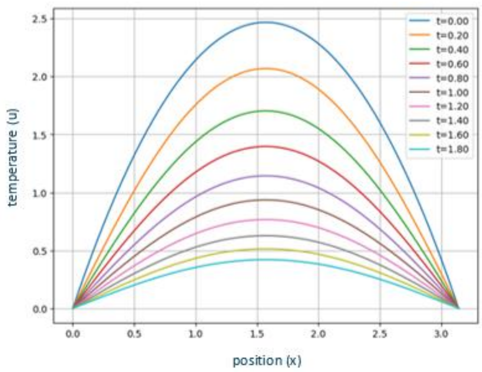

Figure 4 . Heat propagation graph of metal rod by Explicit method with $dt = 0.00001$

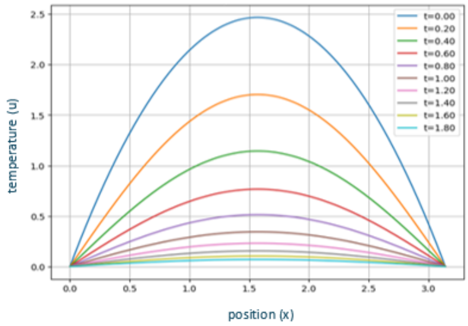

Figure 5. Heat propagation graph of metal rod by Implicit method with $dt = 0.00001$

From the results of this experiment, it can be seen that by reducing the value of dt to 0.00001, the graph of the calculation results by the explicit method becomes more stable and close to the results obtained by the implicit method. This shows that the explicit method is very sensitive to the time interval used. With a smaller time interval, the explicit method can achieve the necessary stability to produce a more accurate solution. However, this also means that to achieve the same stability and accuracy as the implicit method, the explicit method requires much smaller time steps, which can greatly increase the computation time..

Implicit methods, particularly the Crank-Nicolson scheme, exhibit superior stability and consistent accuracy without requiring drastic adjustments to the time interval. This makes them more efficient and reliable to use in heat propagation simulations with given parameters. Overall, although explicit methods can be used with proper stability conditions, implicit schemes such as Crank-Nicolson offer better stability and efficiency advantages, making them more suitable for simulations with larger time steps.

The thermal diffusion process describes how heat spreads through a metal rod from a heated region to a cooler region. Heat tends to flow from high temperatures to low temperatures, causing changes in temperature distribution over time. Insulators at both ends of the rod keep the temperature at zero degrees, which means there is no heat flow in or out of the ends of the rod. The initial temperature $T(x, 0) = x(l - x)$ shows the maximum temperature distribution at the center of the rod, which then spreads across the rod over time.

Therefore, the choice of method largely depends on the specific needs of the simulation. If stability and accuracy in long-term simulations are top priorities, implicit methods are a better choice. However, if short-term simulations or with the need for fast computation per time step are required, the explicit method can be considered, with the caveat that the time step must be small enough to maintain stability. Both methods have their own applications and advantages, and often in practice, both are used according to the context and requirements of the simulation.

Here is a visualization of the graph in three dimensions.

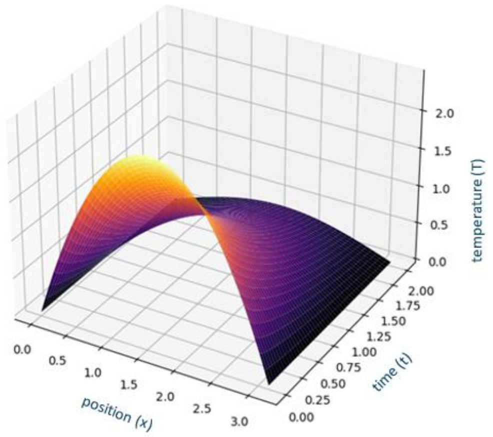

Figure 5. Heat propagation graph of metal rod by Explicit method with $dt = 0.00001$ 3D

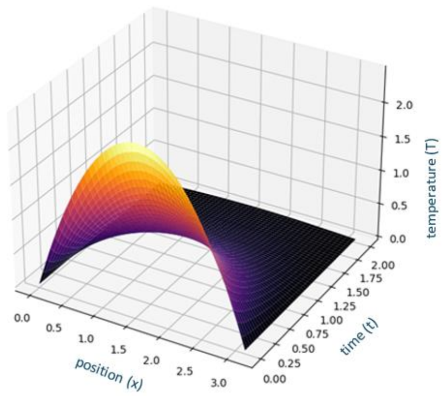

Figure 6. Heat propagation graph of metal rod by Implicit method with $dt = 0.00001$ 3D

At $t = 0$, the initial temperature distribution is parabolic with the highest temperature at the center of the rod and zero temperature at both ends of the rod. Then at time $t > 0$, Heat starts to propagate from the region with higher temperature (center of the rod) to the region with lower temperature (ends of the rod). This is a natural characteristic of the heat diffusion process, where heat tends to move from hot regions to cold regions. The temperature at the center of the rod starts to decrease as the heat spreads outward, while the temperature at the ends of the rod starts to increase as it receives heat from the center of the rod. Finally, at $t = 2$, the temperature distribution along the stem becomes more even. The temperature at the center of the rod continues to decrease as heat continues to propagate towards the ends of the rod, while the temperature at the ends of the rod increases. At the end of the simulation ($t = 2$), the temperature along the rod has approached a more balanced state compared to the initial conditions. There is no longer a sharp temperature gradient along the rod, indicating that the system is approaching thermal equilibrium.

- The resulting graph shows how the temperature distribution changes over time. On each curve, we can see how the temperature at each point along the bar changes.
- The curve at time $t = 0$ shows a parabolic initial temperature distribution.

- The curve at time $t = 2$ shows a more even temperature distribution, with the temperature decreasing at the center of the rod and increasing at the ends of the rod.

This process illustrates the nature of heat diffusion, where heat tends to spread from hot to cold regions, causing a more even temperature distribution over time. This simulation uses the numerical method of Crank-Nicolson scheme which provides a stable and accurate solution to the heat equation. This process describes the nature of heat diffusion, where heat tends to spread from hot regions to cold regions, causing a more even temperature distribution over time. The simulation utilizes the numerical method of Crank-Nicolson scheme which provides a stable and accurate solution to the heat equation.

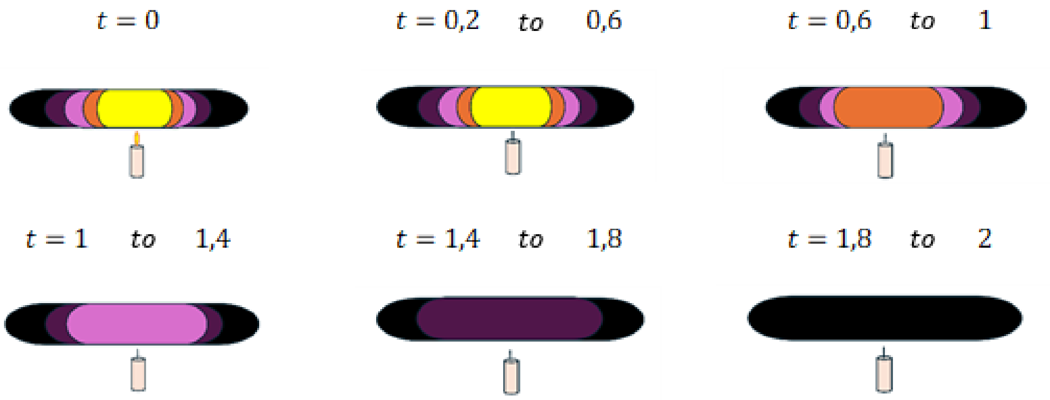

Figure 7. Illustration of heat propagation

Modified

A modification is made by placing a candle just below the position $x = l$, then the candle is lit for some time, after which the candle is turned off. In this case, the temperature change at one end of the metal rod is maintained at zero degrees, the equation given is the same as Equation (5), but the initial value $T(x,0) = x, 0, 0 < x < l$. With the boundary condition $\frac{\partial T(0,t)}{\partial x} = T(l,t) = 0$ for any $t > 0$ and the length of the metal rod as $l = 1$ unit length, the domain $0 \leq x \leq 1$ is obtained. In this case, heat propagation for 1 unit time will be observed, so the domain $0 \leq t \leq 1$ is obtained.

In the explicit method, the graph (see Figure 7) shows how the initial temperature, which varies linearly from 0 at $x = 0$ to 1 at $x = 1$, spreads along the rod over time. The temperature distribution gradually becomes more even, but this method can show larger temperature fluctuations and may experience instability if the parameters are not chosen carefully. In contrast, the Crank-Nicolson method gives a more stable and smooth temperature distribution. The graphs from this method show that the heat propagation is more consistent and less fluctuating, resulting in more realistic and accurate simulations for longer periods of time. Both graphs illustrate how the temperature is evenly distributed along the rod over time after the candle is placed and then extinguished, providing insight into the effectiveness and different characteristics of the two numerical methods.

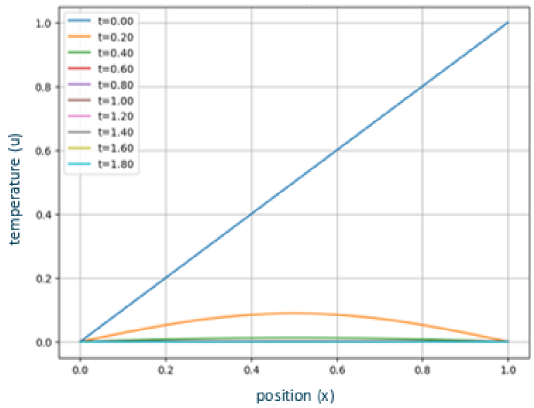

Figure 8. Heat propagation graph of modified metal rod by Explicit method

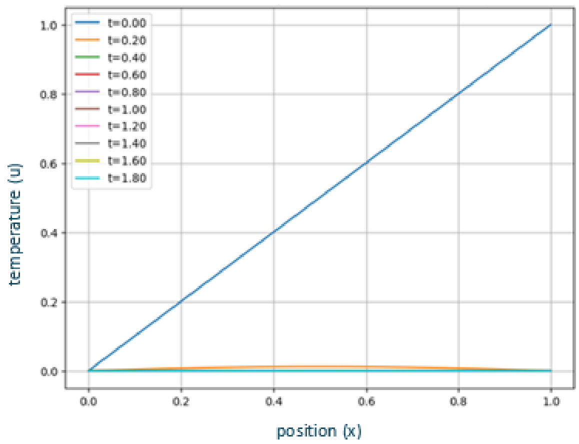

Figure 9. Heat propagation graph of modified metal rod by Implicit method

CONCLUSION

Based on the results of the research and analysis that has been carried out, several important points can be concluded as follows :

Diffusion Process in the Rod

The heat diffusion process modeled through this simulation clearly illustrates how heat propagates from hot to cold regions, causing the temperature along the rod to become more even over time. The use of the Crank-Nicolson scheme in this simulation provides a stable and accurate solution to the heat equation, ensuring that temperature changes can be followed precisely at each time step. This confirms that thermal diffusion is a natural process that seeks to achieve temperature equilibrium in homogeneous systems.

Explicit and Implicit Methods:

The Explicit Method is very sensitive to the size of the time interval used. To achieve good stability and accuracy, these methods require very small time steps, which results in longer computation times, especially for long-term simulations.

The Implicit (Crank-Nicolson) method shows better numerical stability and consistency without requiring drastic adjustment of the time interval. This method remains stable even with larger time intervals, making it more efficient and reliable for heat propagation simulations.

Stability and Accuracy:

Explicit methods are prone to instability and temperature fluctuations if the parameters are not chosen carefully, especially when the value of λ\lambdaλ goes beyond the stability limit.

The Crank-Nicolson method provides a more stable and smooth temperature distribution, resulting in more realistic and accurate simulations, even for longer periods of time.

Computational Efficiency:

Although the explicit method can be used with proper stability conditions, it requires much smaller time steps to achieve the same stability as the implicit method, thus improving the computation time significantly.

The Crank-Nicolson method is more efficient and does not require small time intervals, making it more suitable for simulations with larger time steps and long-term applications..

Experiment Modification:

The modification of placing the candle at a specific position shows how the initial temperature distribution spreads along the rod over time. The Crank-Nicolson method still gives more stable and consistent results than the explicit method under these conditions.

Overall, the choice of method is highly dependent on the specific needs of the simulation. If the main priorities are stability and accuracy in long-term simulations, implicit methods such as Crank-Nicolson are a better choice. However, for short-term simulations or with the need for fast computation per time step, explicit methods can be considered with the caveat that the time step must be small enough to maintain stability. Both methods have their own applications and advantages, and are often used in practice according to the simulation context and requirements. Based on the results of research and data analysis on classroom action research (CAR) which has been carried out for 3 cycles, it is seen that there is an increase in learning outcomes, teacher and student activities, the ability of teachers to manage learning, and good student responses to the application of the PBL model. In the use of this learning method, instructors can make several modifications to elements such as rod length, heat conduction time, and various adjustable variables, so that when applied, students can integrate simulations with the theories and formulas they have learned so far. However, in every learning experience, there are always its own challenges. In this case, the challenge of this learning is the programming algorithms that may encounter errors in certain parts, requiring more time to rebuild the program that will be used.

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