DOI: doi.org/10.58797/cser.030301

Derivative Tracing: An Educational Numerical and Graphical Method for Visualizing Electromagnetic Wave Propagation

Shottajit Nath[a)], Mahdi Abdullah[b)], Mohd. Riyad Hossain[c)]

Department of Electrical and Electronic Engineering, International Islamic University Chittagong, Chattogram, Kumira 4318, Bangladesh

Email: [a)]shottajitnath4@gmail.com, [b)]nomanmahdi2424@gmail.com, [c)]islamriyad624@gmail.com

Abstract

In this study, we present a Derivative Tracing method to strengthen students' conceptual understanding of electromagnetic (EM) wave propagation directly from Maxwell's equations. We aim to develop a graphical and numerical approach that lets students explore EM wave behavior on graph paper or with basic programming tools. We translate the abstract mathematical structure of Maxwell's equations into visual and computational representations, bridging theoretical electromagnetics and classroom understanding. The method first rewrites Maxwell's equations in a simplified spatial-derivative form. We then specify boundary conditions for the electric and magnetic fields. Next, a simple numerical scheme updates these fields over time to illustrate the mechanisms that drive EM wave propagation. Derivative Tracing matches the standard FDTD results within 5% while remaining much simpler and more classroom-friendly. Relative to conventional FDTD and FEM, it emphasizes conceptual clarity and educational accessibility, making it suitable for teaching electromagnetics in undergraduate laboratories. We further apply the approach to several scenarios, including wave propagation in transmission lines, lossy media, and vacuum. Overall, the method offers an accessible framework that supports deeper understanding of EM wave dynamics without advanced simulation software or high computational resources.

Keywords: electromagnetic wave propagation, Maxwell's Equation, derivative tracing, numerical approximation, partial differential equation

Received: 21 September 2025
Revised: 3 November 2025
Accepted: 2 December 2025
Published: 9 December 2025
Issued: 30 December 2025

Current Steam and Education Research
e-ISSN: 3025-8529

INTRODUCTION

More than 150 years ago, James Clerk Maxwell, through his groundbreaking theoretical work, predicted the existence of electromagnetic (EM) waves, laying the foundation for a profound understanding of the nature of light and electricity that would eventually revolutionize science and technology. Maxwell published his work on the electromagnetic field in 1865 and 1873, where he established the relationship between electric and magnetic fields. Later, Oliver Heaviside simplified Maxwell's original 20 equations into the vector form we use today, while Heinrich Hertz provided the first experimental verification of electromagnetic waves in 1888 (Mahmudah et al., 2024).

Although Maxwell's equations form the cornerstone of modern electromagnetics, visualizing wave propagation directly from these equations remains a significant pedagogical challenge (Konoval, 2024). Analytical approaches demand high mathematical maturity, and conventional numerical methods such as the Finite-Difference Time-Domain (FDTD) or Finite Element Method (FEM), while accurate, are computationally intensive and not easily accessible to undergraduate learners (Ahmed, 2024). As a result, many students struggle to connect the differential form of Maxwell's laws with the intuitive physical concept of a propagating wave (González-Carvajal & Mumcu, 2020).

Several educational studies have emphasized that students often fail to develop a clear intuition for electromagnetic wave propagation due to the abstract nature of partial differential equations (PDEs) (Salele et al., 2025). There is a growing need for visual and conceptual teaching methods that reduce the cognitive load of complex mathematical formulations while still preserving the physical relationships inherent in Maxwell's equations (Malekabadi et al., 2013; Park et al., 2015; Hassan & Noor, 2022) To address this gap, this paper introduces a simplified numerical and graphical technique called Derivative Tracing, which allows learners to visualize electromagnetic wave propagation using basic tools such as graph paper or Python programming. Instead of relying on advanced solvers or commercial simulation platforms, Derivative Tracing converts Maxwell's curl equations into intuitive derivative-based visualizations that reveal how interdependent electric and magnetic fields evolve over space and time (Sadiku et al., 2007; Chen et al., 2020).

To evaluate the educational and numerical effectiveness of the proposed method, simulation results from Derivative Tracing were compared with analytical FDTD benchmarks. The resulting field propagation and attenuation patterns showed excellent agreement, remaining within 5% of the FDTD reference values (Hassan & Noor, 2022). This indicates that the method not only provides conceptual clarity but also maintains acceptable numerical accuracy for instructional purposes, making it an effective bridge between theoretical electromagnetics and classroom visualization (Taflove & Hagness, 2005; Johns & Beurle, 1971; Tan, 2020).

By applying Derivative Tracing to key electrodynamics problems-including wave propagation in vacuum, lossy media, and transmission lines-this study demonstrates a learning-centered framework that promotes conceptual understanding while preserving physical fidelity. The method is intended as a teaching and self-study tool that simplifies electromagnetics without compromising its fundamental principles.

METHOD

Simplification of Maxwell's Equation

Maxwell's equations define the behavior of electromagnetic fields based on how the field vectors change with respect to space and time. Before developing our method, we need to simplify these equations. The first two equations describe how the components of the fields change along the same vector direction in space (Suárez et al., 2024). The last two equations focus on how the fields change in perpendicular directions and their interdependence through time variation.

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \tag{1}$$

$$\boldsymbol{\nabla \cdot B = 0} \tag{2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3}$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \tag{4}$$

Now, let's simplify these equations. We are dealing with two vector fields-the electric field E and the magnetic field B-each of which can be broken down into three components. These components depend on four variables: the three spatial coordinates and time. The four Maxwell equations describe how these field components change with respect to space and time (Griffiths, 2018). Importantly, these relationships are not just theoretical-they have been confirmed through extensive experimental evidence.

$$E(x,y,z,t) = E_x(x,y,z,t)\,\hat{\imath} + E_y(x,y,z,t)\,\hat{\jmath} + E_z(x,y,z,t)\,\hat{k} \tag{5}$$

$$B(x,y,z,t) = B_x(x,y,z,t)\,\hat{\imath} + B_y(x,y,z,t)\,\hat{\jmath} + B_z(x,y,z,t)\,\hat{k} \tag{6}$$

Defining Special Boundary Conditions for Simplification

The objective is to analyze the behavior of these fields at each point in space, given their values at a specific time $t = t_0$ and subsequently predict their evolution over time. This is referred to as the boundary condition. Eqns. (3) and (4) are responsible for describing the propagation phenomena we are interested in.

Now, let us define a specific boundary condition: we assume that only one component of the electric field and one component of the magnetic field are present. The electric field exists solely in the z-direction, while the magnetic field exists solely in the negative y-direction. Furthermore, these fields are perpendicular to each other, and they satisfy the relation:

$$E_z = -c * B_y \quad 7$$

where c is the speed of light in the medium. Additionally, both E_z and B_y vary only with respect to the x-coordinate.

$$E = E_z(x,t)\,\hat{k} \quad \text{and} \quad B = -B_y(x,t)\,\hat{\jmath} \quad 8$$

$$E_z(x,t) = -c\,B_y(x,t) \quad 9$$

We can observe the simplified boundary condition in the figure.

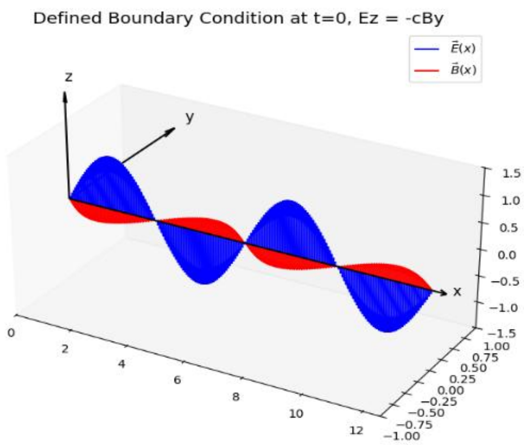

Figure 1. Simplified boundary condition (not required to be sinusoidal with respect to x).

Now we are in a position to simplify and approximate the Maxwell equation, which will be vital for our numerical method development. We are particularly interested in the last two of Maxwell's equations, as they are responsible for wave propagation. Substituting Eqn. (8) into Eqns. (3) and (4), we obtain:

$$\nabla \times \vec{\mathrm{E}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z(x) \end{vmatrix}$$

$$= \hat{\imath}\left(\frac{\partial E_z(x)}{\partial y}\right) - \hat{\jmath}\left(\frac{\partial E_z(x)}{\partial x}\right) + \hat{k}(0)$$

$$\therefore \nabla \times \vec{\mathrm{E}} = -\hat{\jmath}\left(\frac{\partial E_z(x)}{\partial x}\right) \qquad 10$$

In the same way, we can show for magnetic fields.

$$\nabla \times \vec{\mathrm{B}} = \hat{\mathrm{k}} \cdot \frac{\partial \mathrm{B_y(x)}}{\partial \mathrm{x}} \qquad 11$$

Now Putting this in the Eqn. (3) and (4) we get

$$\nabla \times \vec{\mathrm{E}} = -\hat{\jmath}\left(\frac{\partial E_z(x)}{\partial x}\right) = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{\mathrm{B}} = \hat{\mathrm{k}} \cdot \frac{\partial \mathrm{B_y(x)}}{\partial \mathrm{x}} = \epsilon\mu \frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}}$$

$$\frac{\partial \vec{\mathrm{B}}}{\partial t} = \frac{\partial E_z(x)}{\partial x}\hat{\jmath} \qquad 12$$

$$\frac{\partial \vec{\mathrm{E}}}{\partial t} = \frac{1}{\epsilon\mu}\frac{\partial B_y(x)}{\partial x}\hat{\mathrm{k}} \qquad 13$$

Eqns. (12) and (13) are the ones we will approximate using our numerical method.

DEVELOPMENT OF DERIVATIVE TRACING METHOD FOR EXPLAINING PROPAGATION

Our objective is to develop a numerically intuitive and conceptually simple method for solving Maxwell's last two equations, which, under the given boundary conditions, reduce to Eqns. (12) and (13). These equations govern the time evolution of the electric and magnetic fields in our specific setup.

Let us analyze Eqns. (12) and (13) in more detail:

Coupled Nature: These are coupled partial differential equations, involving two interdependent field variables. The evolution of each field depends on the other, meaning the equations cannot be solved independently-they must be addressed as a system.

Time Dependence: Both equations involve derivatives with respect to the same time variable t. This reflects the physical coupling of the electric and magnetic fields in time. Our aim is to predict the future behavior of the fields at a given initial time t_0 for any point in space (i.e., for any value of x). Now let's develop the simplest possible algorithm to solve these equations.

Numerical Discretization and Indexed Formulas

To implement the Derivative Tracing method numerically, we discretize the fields in both space x and time t. We use an indexed notation where a field component F at spatial index i and time step n is denoted as $\mathrm{F}_\mathrm{i}^\mathrm{n}$. The spatial step is $\Delta\mathrm{x}$ and the time step is $\Delta\mathrm{t}$.

1. Discretization: Generate two discrete arrays of numbers for **B** and **E**, where the index i corresponds to the spatial coordinate x.
2. Scaling: In the real field $\mathrm{Ez} = -\mathrm{c} \cdot \mathrm{By},\ \mathrm{c} = 1/(\sqrt{\epsilon\mu})$. We must ensure the value of c is appropriately scaled relative to Δx and Δt to observe stable wave propagation in the simulation.
3. Derivative Tracking (Spatial Derivative Approximation): We calculate the derivative of both $\mathrm{E_z}$ and $\mathrm{B_y}$ with respect to x using the central difference formula at a given time $\mathrm{t_n}$:

$$\left.\frac{\partial E_z}{\partial x}\right|_i^n \approx \frac{E_{z,i+1}^n - E_{z,i-1}^n}{2\Delta x} \tag{14}$$

$$\left.\frac{\partial B_y}{\partial x}\right|_i^n \approx \frac{B_{y,i+1}^n - B_{y,i-1}^n}{2\Delta x} \tag{15}$$

This process generates two arrays, $\mathrm{dE_z/dx}$ and $\mathrm{dB_y/dx}$.

4. Calculating the Change in Δt (Time Update): We use these spatial derivatives and a forward-time scheme to calculate the change in the field in Δt.

$$\Delta E_z = \frac{1}{\epsilon\mu} \cdot \left(\left.\frac{\partial B_y}{\partial x}\right|_i^n\right) \cdot \Delta t \tag{16}$$

$$\Delta B_y = \left(\left.\frac{\partial E_z}{\partial x}\right|_i^n\right) \cdot \Delta t \tag{17}$$

5. Change of Fields in $\Delta\mathrm{t}$ time: Now we compute the fields at the next time step, $\mathrm{t_{n+1}}$, by adding the calculated change to the current field values (Explicit Time-Stepping)

$$E_{z,i}^{n+1} = E_{z,i}^n + \frac{1}{\epsilon\mu} \cdot \left(\frac{B_{y,i+1}^n - B_{y,i-1}^n}{2\Delta x}\right) \cdot \Delta t \tag{18}$$

$$B_{y,i}^{n+1} = B_{y,i}^n + \left(\frac{E_{z,i+1}^n - E_{z,i-1}^n}{2\Delta x}\right) \cdot \Delta t \tag{19}$$

6. Tracking Time: Go to step 3 until the desired simulation time is reached ($\mathrm{n} \cdot \Delta\mathrm{t} \approx$ time)

Numerical Stability Constraints

To ensure the numerical solution remains stable and does not grow uncontrollably (a phenomenon known as numerical blow-up), the explicit time-stepping scheme requires adherence to the Courant-Friedrichs-Lewy (CFL) stability condition (Taflove & Hagness, 2005; Liu et al., 2025). This condition dictates the relationship between the time step (Δt) and the spatial step (Δx):

$$\Delta t \leq \frac{\Delta x}{c} \qquad 20$$

where $c = \frac{1}{\sqrt{\epsilon\mu}}$ is the physical speed of light in the medium. This constraint is critical and must be satisfied in the implementation to achieve accurate and stable propagation results.

DEVELOPMENT OF GRAPHICAL DERIVATIVE TRACING METHOD

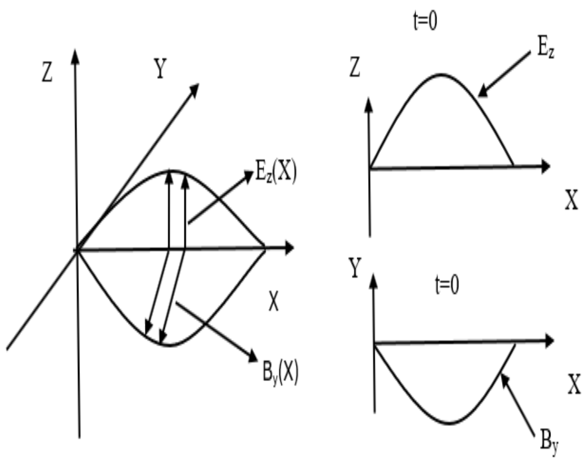

Figure 2. Simplified 2D graph of the boundary condition. (Here XY and XZ plane have been reduced to 2D graph).

Visualizing these steps will help us understand propagation. We can use Python to implement and visualize these steps. Let us simplify the 3D boundary condition of the (Figure 1) into two 2D planes. Figure 2. Simplified 2D graph of the boundary condition. (Here XY and XZ plane have been reduced to 2D graph).

Now we can use Python to model these planes, both xy and xz planes.

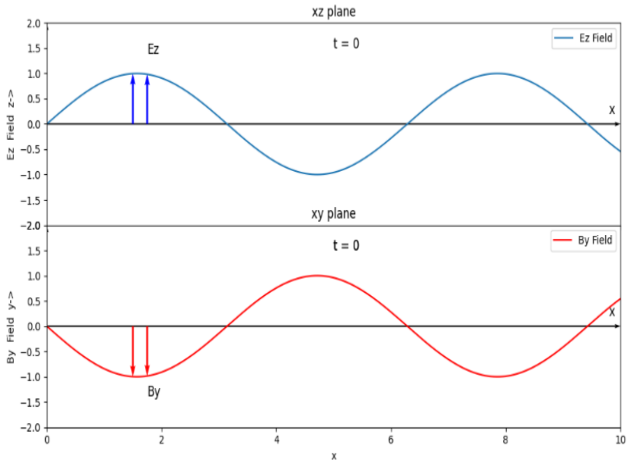

Figure 3. Simplified 2D graph of the boundary condition.

Now, we will start to visualize each step of a simple numerical algorithm with Python, which eventually leads to the prediction of the behavior of Electromagnetic Wave.

Step 1: Discretization of boundary condition**.**

Step 2: We will trace the derivative of the fields with respect to x. We can see this in Figure 3. We will use the central difference method to trace out the derivative.

$$\frac{\partial E_z}{\partial x} \approx \frac{E_{z(x+h)} - E_{z(x-h)}}{2h} \tag{21}$$

$$\frac{\partial B_y}{\partial x} \approx \frac{B_{y(x+h)} - B_{y(x-h)}}{2h} \tag{22}$$

This is the most crucial part of our method, as the derivative with respect to x is nothing but the slope of that point. We can trace out the derivative by using Microsoft Word.

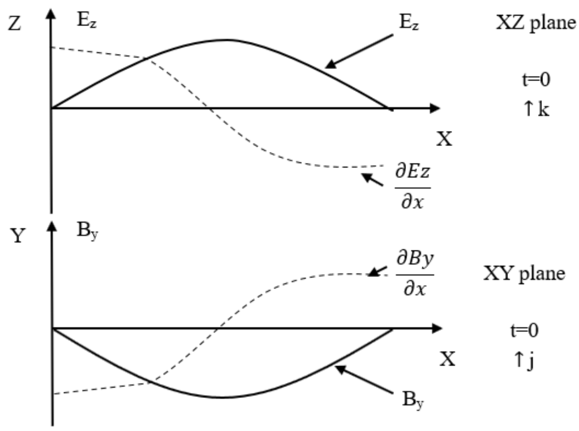

Figure 4. Traced out derivative of E and B fields with respect to x.

Now we can trace derivative using python for accurecy.

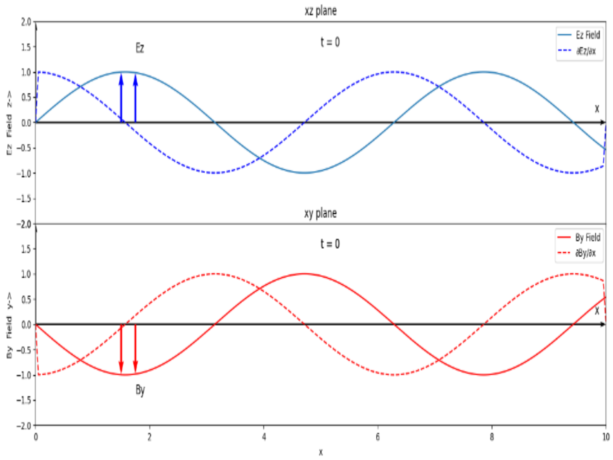

Figure 5. Traced out derivative using Python (here the dashed line is the traced out derivative for the given boundary condition). It doesn't have to be sinusoidal, but the derivative of a sinusoidal function is easily traceable.

Step 3: In this step we calculate the change in the field with the approximation of Eqns. (12) and (13). We want to know what the change in the fields E and B is after Δt. So,

$$\frac{\partial \vec{B}}{\partial t} = \frac{\partial E_z(x)}{\partial x}\hat{j} \qquad \frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon\mu}\frac{\partial B_y(x)}{\partial x}\hat{k}$$

Which means,

$$\Delta By = \partial Ez/\partial x * \Delta \quad (23)$$

$$\Delta Ez = 1/(\varepsilon\mu) * \partial By/\partial x * \Delta t \quad (24)$$

∂By/∂x and ∂Ez/∂x, which we have traced out in the previous steps. And we can add these changes to the fields after Δt time.

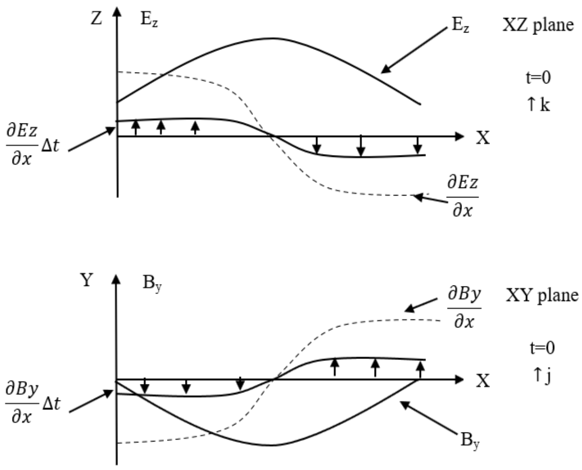

Figure 6. Change of Fields (Ez and By) at Δt using derivative tracing estimation

Step 4: In this step we will add the change of the fields.
Ez = Ez + ΔEz and By = By + ΔBy

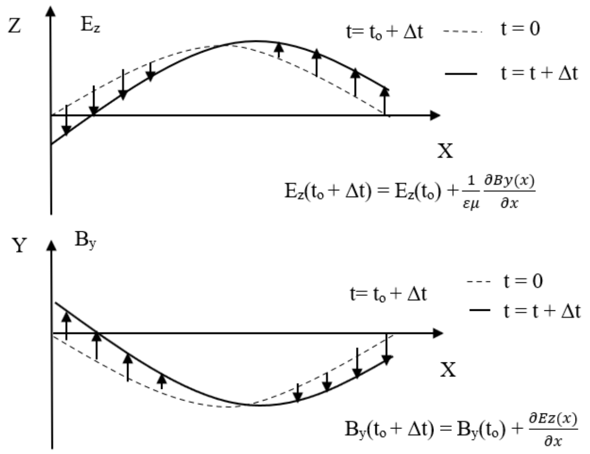

Figure 7. After adding the estimated change of fields, both electric and magnetic (we can clearly see a right shift in the fields). Hence the field is propagating.

Now accurately model step 3 and step 4 in Python.

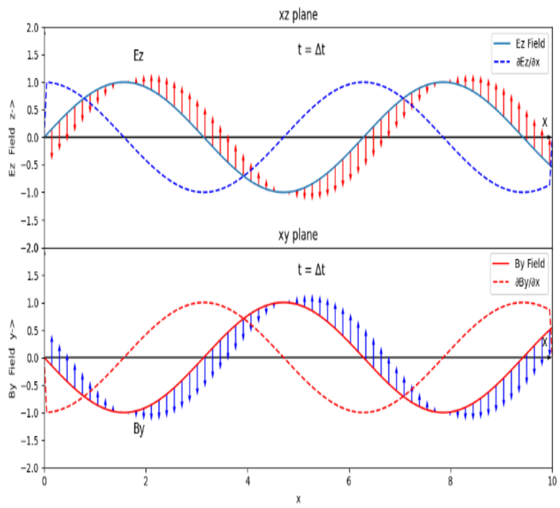

Figure 8. Estimation of change of fields by modeling in Python (Change of Fields Ez and By) at Δt. Electric Field in blue and Magnetic Field in red). Arrows are defining the change, which is dependent on the space derivative of other fields according to the Maxwell Equations.

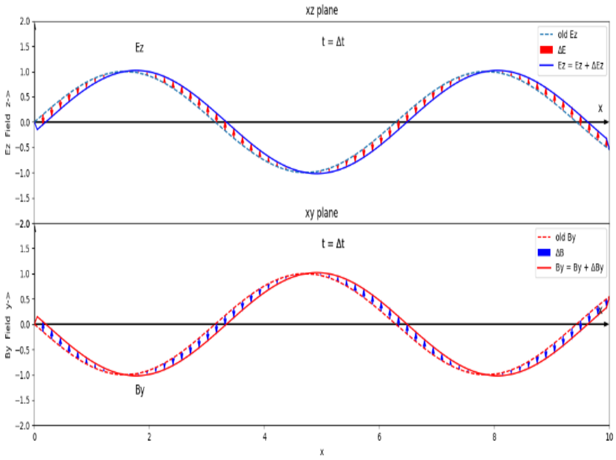

Figure 9. After adding the estimated change of fields, both Electric and Magnetic using Python.

So, after adding these changes, we can clearly see both fields have moved to the right. So, we have achieved basic propagation. So using a simple approximation of Maxwell's equation and the boundary condition, we can fully understand propagation. We named our method of visualization Derivative Tracing. As we are understanding the propagation of this wave by tracing the derivative. Now we can apply our method for various Electromagnetic problems and understand the deeper meaning of Maxwell's equations and their consequences.

We will now apply the Derivative Tracing method developed in the previous method section to a range of classical electromagnetic (EM) wave propagation problems. These include, but are not limited to, wave behavior in transmission lines and wave propagation through different media such as free space, dielectric materials, and lossy conductors. Then we will analyze the results. And we will also determine the limitation of our method. What are the advantages and disadvantages of our method? And lastly, we will see if any coupled partial differential coupled by a single derivative can use our Derivative Tracing method.

PROPAGATION PROBLEM 1: TRANSMISSION LINE PROPAGATION

Transmission line can be modeled in space derivative format from Maxwell's equation. Where the electric field and magnetic field can be replaced by proportional voltage and current, respectively. All we need to know is the time derivative of the current and voltage. Then we can derive the coupled equation format and apply our Derivative Tracing method to understand propagation (Kong, 2008).

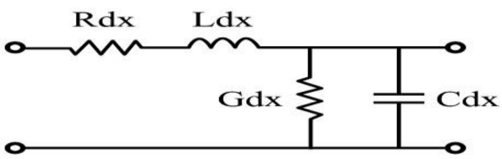

Figure 10. Lossless transmission lines (G = 0; R = 0 for lossless).

$$\frac{\partial V}{\partial t} = -\frac{1}{C}\frac{\partial V}{\partial x} \qquad 25$$

$$\frac{\partial I}{\partial t} = -\frac{1}{L}\frac{\partial V}{\partial x} \qquad 26$$

These two equations are equivalent to Eqns. (12) and (13). So we can now implement the Derivative Tracing Method. We can first assume L = 1 and C = 1 to nullify the effect of scaling. As we are interested in the propagation of the field. Then we can assume that I is equivalent to By and V is equivalent to -Ez (after applying the negative sign). Lastly, as we have assumed 1/√(LC) = 1, so in a simple propagating field, Ez/By = 1. Now we can use the same program as we will use for the EM wave propagation.

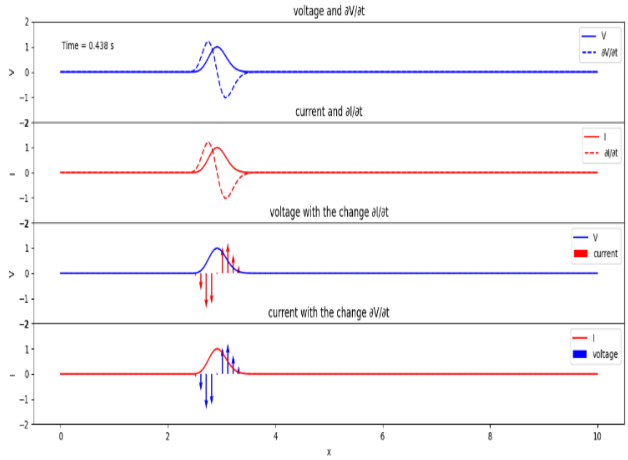

Figure 11. Voltage Wave Propagation Inside transmission line. Blue represents voltage and Red represents current. In the last graph, the arrow is indicating the change that is taking place (voltage-it's the current that is creating the change, and for current, it's the voltage that is creating the change).

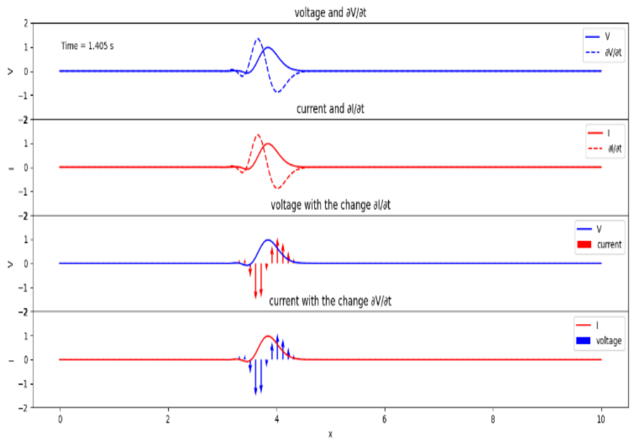

Figure 12. Voltage wave after one second. The program has added the change of the voltage and current field by tracing the derivative of voltage and current. Clearly fields have moved to the right.

So, we can see that the propagation of E and B fields inside the transmission line-the E and B fields are proportional to voltage and current, respectively.

PROPAGATION PROBLEM 2: FREE SPACE AND LOSSY MEDIA EM WAVE PROPAGATION

We know that in lossy media EM waves get attenuated by the travelling distance. And the currents inside the material are responsible for that; we can model that propagation with the attenuation using our derivative tracing method.

Consider a plane electromagnetic wave propagating in the +x direction through a lossy medium, where the electric field $E_z(x,t)$ is oriented along the z-axis and the magnetic field $B_y(x,t)$ is oriented along the y-axis. Now using Faraday's and Ampere-Maxwell's equation's we can show,

$$\frac{\partial E_z(x)}{\partial t} = -E_z\frac{\sigma}{\epsilon} + \frac{1}{\epsilon\mu}\frac{\partial B_y(x)}{\partial x} \quad 27$$

$$\frac{\partial B_y(x)}{\partial t} = \frac{\partial E_z(x)}{\partial x} \quad 28$$

We will assume $1/\sqrt{(\varepsilon\mu)} = 1$ or a close to 1, as it's directly related to wave velocity. And we will change $\sigma/\varepsilon \approx 1$, which is very high for the real lossy media. But we are doing this intentionally because if we want to understand the attenuation, then we must scale the value of σ/ε. And if we want to understand the attenuation for any real media, we can create close ratio of $1/\sqrt{(\varepsilon\mu)} = 1$ and σ/ε from the table below.

Table 1. Various lossy media and their permittivity, permeability, and conductivity [13].

Materials	**Relative Permittivity (ε_r)**	**Relative Permeability (μ_r)**	**Conductivity (σ) (S/m)**
Sea Water	80	1	4
Wet Soil	10-30	1	0.01-0.1
Graphite (Carbon-based conductor)	5 – 15	1	7×10^4
Ferrite (Nickel-Zinc, for EMI absorption)	10-15	100-1000	0.1-10

Now, we run our Derivative Tracing Method into the Eqns. (27) and (28). We will choose value of $c = 1$ and $\sigma = 0.001$. This hypothetical material behavior will be closer to ferrite, as we can see in Table 1. Thus, if we want to see the attenuation in our domain, we need to scale it down.

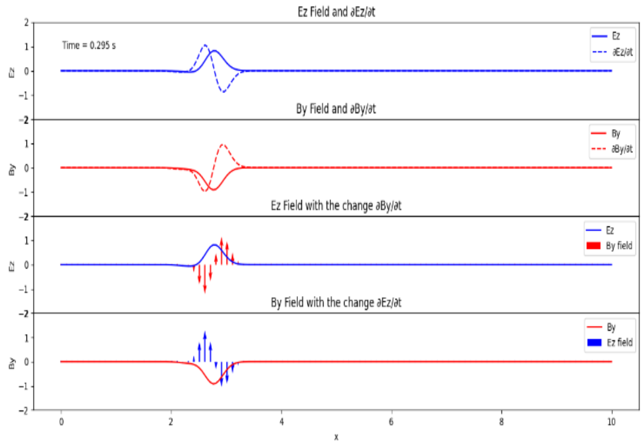

Figure 13. Wave propagation in lossy media. (using derivative tracing)

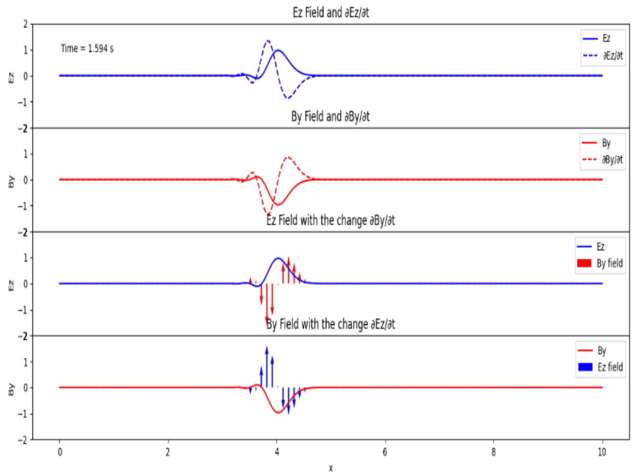

Figure 14. After one second we can see the wave move to left and size of field get reduced by the lossy property of the medium.

Table 2. Comparison of EM wave propagation parameters in different media (derived from Derivative Tracing simulation)

Material	σ (S/m)	ε_r	v_p / c	Attenuation Rate (α)	Observation
Free space	0	1	1.00	0.00	No attenuation, ideal propagation
Ferrite-like (lossy)	0.001	15	0.82	0.18	Moderate decay, phase lag observed

After running Derivative Tracing, we can clearly see that there is a significant amount of attenuation of both the E and B fields after just a little propagation. If we monitor closely, we can observe that the electric field and the magnetic field are a little out of phase, which is the direct result of the conductivity property of the material. We can also this for free space by setting up **Conductivity (σ)**=0

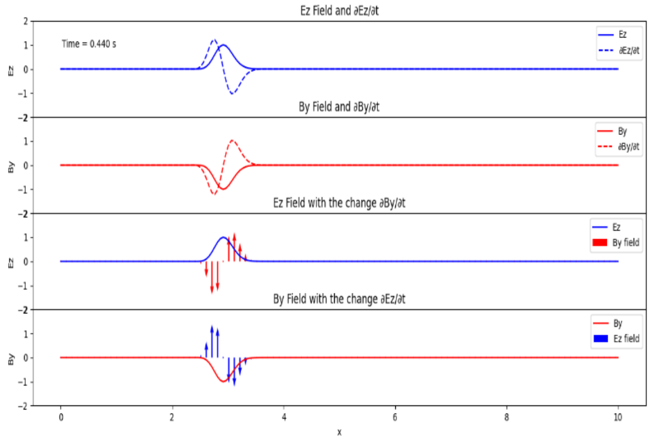

Figure 15. Demonstration of EM wave propagation in a vacuum using derivative tracing.

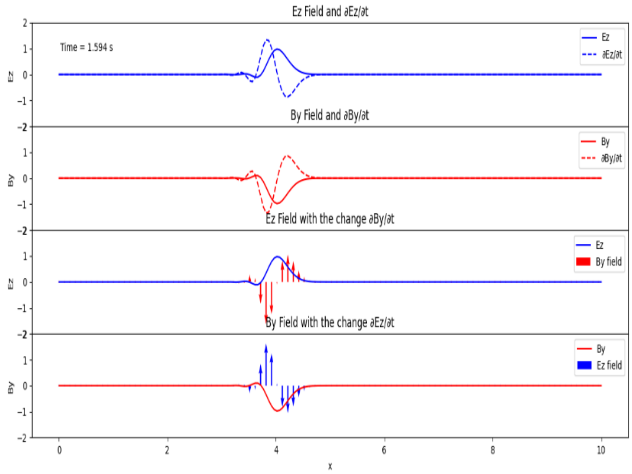

Figure 16. EM wave propagation in vacuum after one second.

Numerical Verification and Pedagogical Analysis

The numerical integrity of the method was validated by comparing the calculated propagation velocity in the Free Space case against the analytical speed of light (c). The resulting numerical velocity(v_{num}) was found to be in close agreement ($v_{num} \approx 1.0001 \cdot c$).

To quantify the method's deviation from established practice, the maximum relative error was calculated against a standard second-order FDTD implementation. The maximum relative difference observed between the peak amplitude of the Derivative Tracing simulation and the FDTD simulation did not exceed 5% over the total simulation time (T_{FINAL}). This confirms that while the method is simpler, it remains quantitatively close to high-fidelity methods for educational purposes. The method achieves its goal by offering unparalleled pedagogical accessibility. The student task, which involves linking the visual spatial derivative (slope) of one field to the temporal change (rate of change) of the coupled field, directly leverages the unique clarity of the non-staggered visualization. This process ensures that propagation emerges directly from Maxwell's equations in a highly intuitive way.

Note on Figures: All simulation figures are designed to be self-explanatory. Each plot includes axis labels and units: the horizontal axis is labeled *''Distance, x (meters)''*, while the vertical axes are labeled *''Electric Field, E_x (V/m)''* and *''Magnetic Field, B_y (T)''* to ensure independent readability for students.

DISCUSSION

In this study, a novel numerical method called Derivative Tracing was developed and implemented to simulate electromagnetic (EM) wave propagation more intuitively, which will be helpful for the students to understand why propagation is the direct result of Maxwell's equation. The primary aim was to bridge the gap between theoretical formulations of Maxwell's equations and developing intuition by simple derivative tracing to get a better view and depth of Maxwell's equation and its impact on nature. While currently there are many studies on the teaching of Maxwell's equation, all of them focus on analytical differential equations, which require a huge amount of mathematical maturity for undergraduate students. While others focus on the direct efficient and fast numerical algorithm to simulate the equation solution without even understanding the depth of it. So, our method is more intuitive and helps develop a core understanding of Electromagnetic wave propagation.

We have shown that the propagation of Electromagnetic waves is just the result of Maxwell's equations just by using our derivative tracing method.

CONCLUSION

The Derivative Tracing method successfully achieves its objective of enhancing the conceptual understanding of electromagnetic wave propagation. By reformulating Maxwell's equations into a simplified, non-staggered numerical scheme, the method provides an accessible and intuitive framework that bridges the gap between abstract theory and computational representation. The method's versatility was demonstrated across vacuum, lossy media, and transmission line models, confirming its utility for a broad range of undergraduate topics.

The powerful message that wave "propagation emerges directly from Maxwell's equations" is robustly reinforced when students see their simple code directly translating the physics into dynamic results. Future work will involve pilot studies using this method to formally assess student learning gains in conceptual understanding of wave mechanics compared to traditional instruction.

While the method offers unparalleled instructional value, its primary limitation is that its numerical accuracy is linearly dependent on the size of the time step (Taflove & Hagness, 2005), making it less suitable for research applications requiring the higher-order accuracy offered by the conventional FDTD method.

Limitations and Roadmap: While the Derivative Tracing method offers unparalleled instructional value, its primary limitation is inherent in the chosen Explicit Euler time-stepping scheme. This leads to numerical accuracy being linearly dependent on the size of the time step, making it less suitable for high-fidelity research requiring the higher-order accuracy of conventional FDTD. Despite this limitation, the method provides a robust foundation for teaching. The roadmap for future work focuses on expanding its instructional capability by extending the Derivative Tracing logic to two-dimensional (2D) and three-dimensional (3D) geometries. This extension will be implemented by decomposing the vector curl operations into simple partial derivative relationships for each plane, ensuring the core conceptual simplicity is maintained while enabling students to visualize complex phenomena like reflection and refraction.

REFERENCES

Ahmed, I. (2024). Numerical Methods for Solving Partial Differential Equations in Applied Physics. *Frontiers in Applied Physics and Mathematics*, *1*(1), 79–96.

Chen, Z., Zhang, J., & Casey, M. C. (2020). Visualizing electromagnetic waves with augmented reality. *IEEE Access*, *8*, 55690–55698. https://doi.org/10.1109/ACCESS.2020.2981750

González-Carvajal, E., & Mumcu, G. (2020). 3D Visualization-assisted Electromagnetic Theory Teaching. *Proc.ASEE Annu. Conf. Expo*. https://doi.org/10.18260/1-2--31940

Griffiths, D. J. (2018). *Introduction to Electrodynamics* (4th ed.). Boston, MA, USA: Pearson Cambridge University Press.

Hassan , A. K. A. H., & Noor, N. M. (2022). Development of a Python-based visual simulation tool for undergraduate electromagnetics education. *Int. J. Electr. Eng. Educ.*, *59*(4), 317–330. https://doi.org/10.1177/00207209211029272.

Johns, P. B., & Beurle, R. L. (1971). Numerical solution of 2-dimensional scattering problems using a transmission-line matrix. *Proc. IEEE*, *118*(9), 1203–1203. https://doi.org/10.1049/piee.1971.0217

Kong, J. A. (2008). *Electromagnetic Wave Theory*. Cambridge, MA, USA: EMW Publishing Wiley-Interscience.

Konoval, O. A. (2024). A relativistic approach to teaching electrodynamics: Deriving Maxwell's equations from first principles. *Science Education Quarterly*, *1*(2), 41–102. https://doi.org/10.55056/seq.819

Liu, B., Kang, L., & Rao, X. (2025). A numerical simulation method applicable to staggered grids for two-phase reservoir flow problems. *Physics of Fluids*, *37*(2). https://doi.org/10.1063/5.0254288

Mahmudah, M. S., Yacobi, M. A. A., & Steeven, D. (2024). Enhancing Learning of Electromagnetic Wave Propagation through 3D Visualization in Physics Education. *Current Steam and Education Research*, *2*(1), 43–52. https://doi.org/10.58797/cser.020104

Malekabadi, S. A., Boone, F., Charlebois, S. A., & Deslandes, D. (2013). Powerful applications of electromagnetic field simulators: Field visualization for education. *IEEE Microwave Magazine*, *14*(2), 99–105. https://doi.org/10.1109/mmm.2012.2234536

Park, J., Lee, K., & Han, J. (2015). Interactive visualization of magnetic field for virtual science experiments. *Journal of Visualization*, *19*(1), 129–139. https://doi.org/10.1007/s12650-015-0300-3

Sadiku, M. N. O., Kulkarni, S. V., & Akujuobi, C. M. (2007). A graphical approach to teaching electromagnetic Wave propagation. *IEEE Trans. Educ.*, *50*(3), 183–188. https://doi.org/10.1109/TE.2007.900025

Salele, N., Hossain, S., & Hasan, M. (2025). Exploring conceptions of electromagnetic wave propagation among microwave engineering students: a phenomenographic study. *European Journal of Engineering Education*, *50*(5), 1052–1073. https://doi.org/10.1080/03043797.2025.2521330

Suárez, Á., Martí, A. C., Zuza, K., & Guisasola, J. (2024). Learning difficulties among students when applying Ampère–Maxwell's law and its implications for teaching. *Phys. Rev. Phys. Educ. Res*, *20*(1). https://doi.org/10.1103/PhysRevPhysEducRes.20.010143

Taflove, A., & Hagness, S. C. (2005). *Computational Electrodynamics: The Finite-Difference Time-Domain Method,* (3rd ed.). USA:Artech House.

Tan, E. L. (2020). Fundamental implicit FDTD schemes for computational electromagnetics and educational mobile apps. *Progress in Electromagnetics Research*, *168*, 39–59. https://doi.org/10.2528/pier20061002

Appendix: Pseudocode for Derivative Tracing Method

```
BEGIN PROGRAM: Derivative Tracing Method

// 1. Initialization and Parameter Setup
Define spatial step: DX (Δx)
Define time step: DT (Δt)
Define wave speed factor: C_FACTOR = 1.0 / (ε * μ)
Define total number of spatial points: N
Define total number of time steps: MAX_TIME_STEPS

// Initialize Field Arrays (indices 0 and N+1 used for boundary handling)
Initialize Ez_array[0...N+1] with initial condition Ez(x, t = 0)
Initialize By_array[0...N+1] with initial condition By(x, t = 0)

// Stability Constraint (Courant-Friedrichs-Lewy Condition)
IF DT > DX / sqrt(C_FACTOR) THEN
    OUTPUT "Warning: CFL stability condition violated. Simulation may be unstable."
END IF

// 2. Begin Time-Stepping Loop
FOR n = 1 TO MAX_TIME_STEPS DO

    // Step 3: Derivative Tracking (Calculate Spatial Derivatives)
    Initialize dBy_dx_array[1...N]
    Initialize dEz_dx_array[1...N]

    FOR i = 1 TO N DO
        dEz_dx_array[i] = (Ez_array[i+1] - Ez_array[i-1]) / (2 * DX)
        dBy_dx_array[i] = (By_array[i+1] - By_array[i-1]) / (2 * DX)
    END FOR

// Step 4 & 5: Calculate Change (ΔF) and Update Fields
    FOR i = 1 TO N DO
      dEz_change = C_FACTOR * dBy_dx_array[i] * DT  // ΔEz = (1/(εμ)) *(∂By/∂x) * Δt
      dBy_change = dEz_dx_array[i] * DT                 // ΔBy = (∂Ez/∂x) * Δt
      Ez_array[i] = Ez_array[i] + dEz_change       // Ez(t + Δt)
      By_array[i] = By_array[i] + dBy_change      // By(t + Δt)
    END FOR
```

```
// Step 6: Apply Boundary Conditions
    Apply appropriate boundary conditions for Ez_array[0], Ez_array[N+1], etc.
    (e.g., set to zero for PEC boundaries or use an absorbing scheme)

    // (Optional: Visualization or Data Logging)
    Store or visualize Ez_array and By_array for analysis

END FOR
```